# Multiband Pi-shaped Structure with Resonators for Tri-band Wilkinson Power Divider and Tri-band Rat-Race Coupler

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***Abstract* — This paper presents a novel multiband Pi-shaped structure with resonators. By employing resonators in conventional Pi-shaped structure, multiband quarter wavelength transmission line is achieved. In order to demonstrate our proposed multiband structure, we designed and fabricated tri-band Wilkinson power divider and tri-band rat-race coupler. We can reduce the size by sharing the stub with resonators of two adjacent Pi-shaped structures and making stubs inside the two demonstrated circuits. Measured results show excellent performance comparing to the classical component at single frequency.**

***Index Terms* — Multiband, Pi-shaped structure, resonator, Wilkinson power divider, rat-race coupler.**

## I. INTRODUCTION

Interoperability and co-existence between multi standards is the main issue in modern communication systems. To satisfy this harsh requirement, multi-band radio systems are used at different frequencies associated with different standards.

To meet the trend of multiband radio system, it is required to develop new type of multiband component such as power divider, rat-race coupler, and power amplifier. In past years, many researches for multiband applications have been presented. For example, coupled line [1] has been applied effectively in dual-band Wilkinson power divider; Stepped-impedance microstrip line is employed for multiband rat-race coupler [2]; Resonator has been used for dual-band matching network [3] and dual-band Wilkinson power divider design [4]. Our objective is to design multiband components by using the proposed multiband Pi-shaped structure with resonators and to reduce the size of the proposed multiband components.

Section II will present the topology and principle of the proposed multiband quarter wavelength transmission line for infinite target bands. A tri-band Wilkinson power divider and tri-band rat-race coupler using the proposed multiband Pi-shaped structure with resonators have been demonstrated in section III.

## II. ANALYSIS OF MULTIBAND PI-SHAPED STRUCTURE

The topology of our proposed Pi-shaped multiband structure is shown in Fig. 1. Parallel resonator $f_x$ (x=1, ... , n-1) used in the proposed Pi-shaped circuit blocks signal generated at frequency $f_x$ (x=1, ... , n-1), where $f_1 > \ldots > f_{n-1}$.

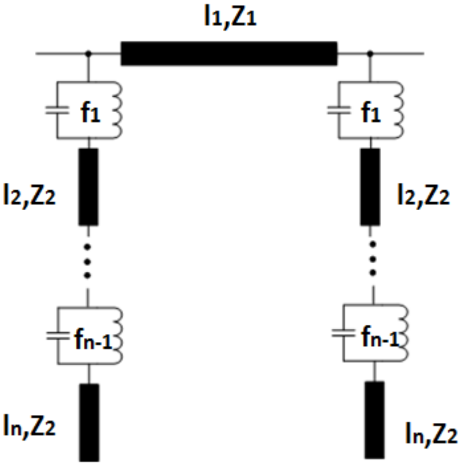


Fig. 1. Multiband Pi-shaped structure with resonators.

Fig. 2 shows the principle of our proposed Pi-shaped structure. At frequency $f_x$ (x=1, … , n-1), signal is blocked by the resonator $f_x$, and passing through resonator $f_y$ (y=0, …, x-1), where the resonator $f_0$ means there's no resonator. The proposed Pi-shaped structure becomes the quarter wavelength transmission line at the corresponding frequency.

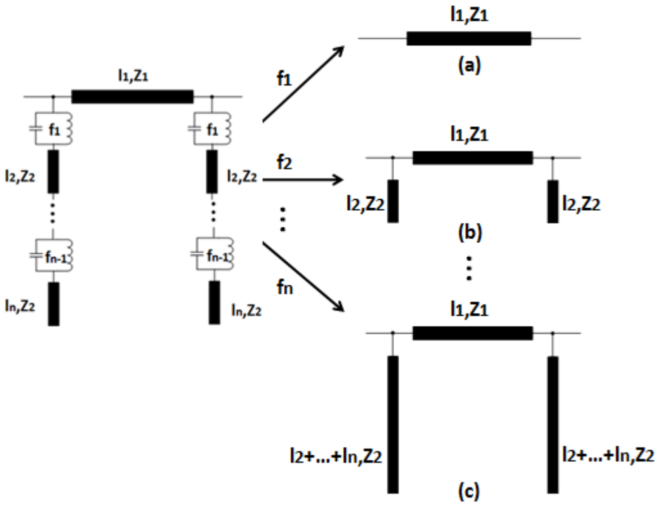


Fig. 2. Principle of multiband Pi-shaped structure.

For example, at frequency $f_1$, signal is blocked by the first resonator $f_1$, the initial proposed structure (Fig. 1) becomes a transmission line (Fig. 2(a)). At frequency $f_2$, because the signal can go through the first resonator and be blocked at the second resonator, our proposed multiband Pi-shaped structure (Fig. 1) transforms to Pi-shaped line with impedance $Z_2$ and length $l_2$ (Fig. 2(b)). Finally, the same operation is repeated at frequency $f_n$ ($n \geq 2$), the first stub joins to the (n-1)th stub to be considered as one longer open-circuit stub (Fig. 2(c)).

In order to calculate the length and impedance of stubs we used ABCD matrix which allows simplifying the calculation [5]. At frequency $f_n$ ($n \geq 2$), Fig. 2(c) with symmetrical structure ($l_2+\ldots+l_n$, $Z_2$) at both ends of $l_1$ corresponds to the matrix below:

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY_2 \tan\beta_n(l_2+\ldots+l_n) & 1 \end{bmatrix} * \begin{bmatrix} \cos\beta_n l_1 & jZ_1 \sin\beta_n l_1 \\ jY_1 \sin\beta_n l_1 & \cos\beta_n l_1 \end{bmatrix} * \begin{bmatrix} 1 & 0 \\ jY_2 \tan\beta_n(l_2+\ldots+l_n) & 1 \end{bmatrix} \tag{1}$$

Where $\beta_n = \dfrac{2\pi}{\lambda_n}, Z_1 = 70.7\Omega, l_1 = \dfrac{\lambda_1}{4}$.

At $f_n$, ABCD matrix of a quarter wavelength transmission line can be expressed as:

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \begin{bmatrix} 0 & jZ_{0n} \\ jY_{0n} & 0 \end{bmatrix} \tag{2}$$

Ideally the equivalent length of the proposed Pi-shaped circuit should be $\lambda_n / 4$ at frequency $f_n$. To do that, we equalize the equations (1) and (2). We can deduce:

$$A_n = \cos\beta_n l_1 - Y_2 Z_1 \tan\beta_n(l_2+\ldots+l_n)\sin\beta_n l_1 = 0$$

$$\Rightarrow Z_1 \tan\beta_n l_1 = Z_2 \cot\beta_n(l_2+\ldots+l_n) \tag{3}$$

$$B_n = jZ_1 \sin\beta_n l_1 = jZ_{0n}$$

$$\Rightarrow Z_{0n} = Z_1 \sin\beta_n l_1 \tag{4}$$

$$\begin{aligned} C_n &= jY_2 \tan\beta_n(l_2+\ldots+l_n)\cos\beta_n l_1 + jY_1 \sin\beta_n l_1 \\ &- jY_2 Y_2 Z_1 \tan\beta_n(l_2+\ldots+l_n)\tan\beta_n(l_2+\ldots+l_n) * \\ &\sin\beta_n l_1 + jY_2 \tan\beta_n(l_2+\ldots+l_n)\cos\beta_n l_1 = jY_{0n} \end{aligned} \tag{5}$$

$$D_n = \cos\beta_n l_1 - Y_2 Z_1 \tan\beta_n(l_2+\ldots+l_n)\sin\beta_n l_1 = 0$$

$$\Rightarrow Z_1 \tan\beta_n l_1 = Z_2 \cot\beta_n(l_2+\ldots+l_n) \tag{6}$$

We can deduce:

$$\Rightarrow Z_1 \tan\beta_n l_1 = Z_2 \cot\beta_n(l_2+\ldots+l_n) \tag{7}$$

$$\Rightarrow Z_{0n} = Z_1 \sin\beta_n l_1 \tag{8}$$

In equation (7), by giving $Z_2 = R\,\Omega$, we can calculate the stub length ($l_2+\ldots+l_n$) ($n \geq 2$). To satisfy $Z_{0n} = 70.7\,\Omega$, we can make a trade-off with $Z_1$ to obtain the best performance of multiband operation.

## III. APPLICATION OF MULTIBAND PI-SHAPED STRUCTURE

### A. Wilkinson Power Divider

By employing the multiband Pi-shaped structure proposed in section II, we fabricated one tri-band Wilkinson power divider to demonstrate our proposed method. The topology and fabricated circuit of our tri-band Wilkinson power divider are shown in Fig. 3. The substrate we used is TLX-8 of Taconic with dielectric constant 2.55.

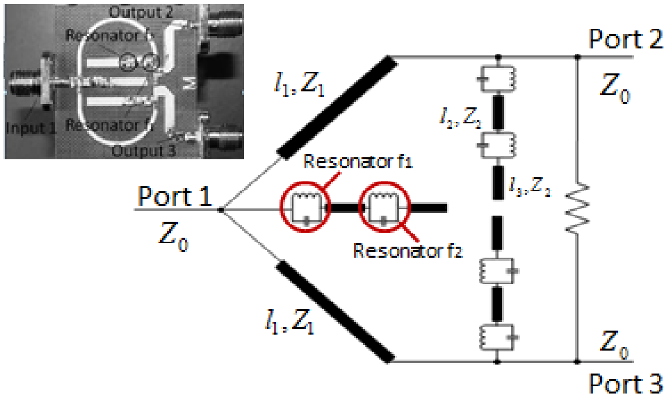


Fig. 3. Topology of proposed tri-band Wilkinson power divider.

The simulated and measured results are shown in Fig. 4. For measured results, at 1/1.5 /2.5 GHz, input and output return loss is more than 12.2 dB, insertion loss is better than 3.8 dB and output isolation is better than 18.8 dB for three frequencies. Simulated results are found to be consistent with measured results. As shown in Fig. 7, the phase difference between two output ports is less than 2°.

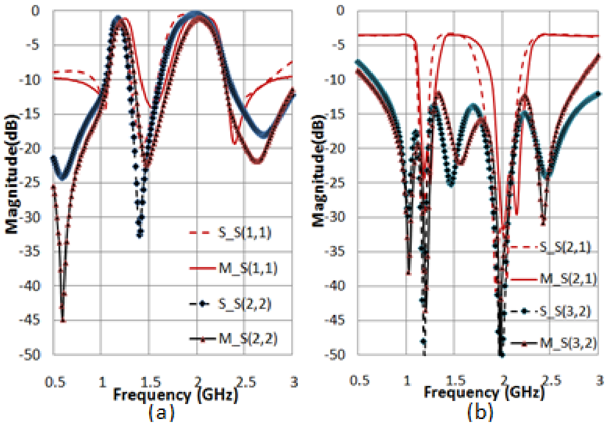


Fig. 4. Simulated (S) and measured (M) results of proposed tri-band Wilkinson power divider.

### *B. Rat-Race Coupler*

We designed and fabricated one tri-band rat-race coupler as shown in Fig. 5 with the same substrate used in tri-band Wilkinson power divider.

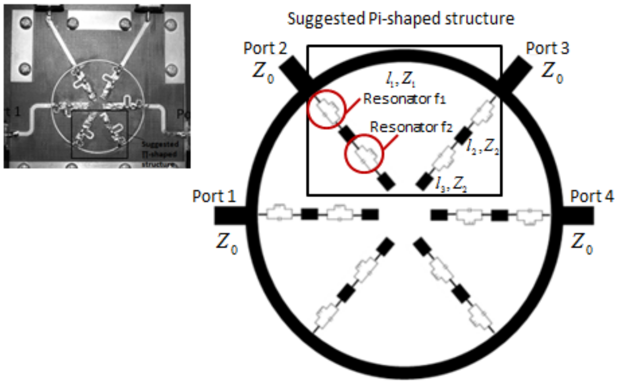
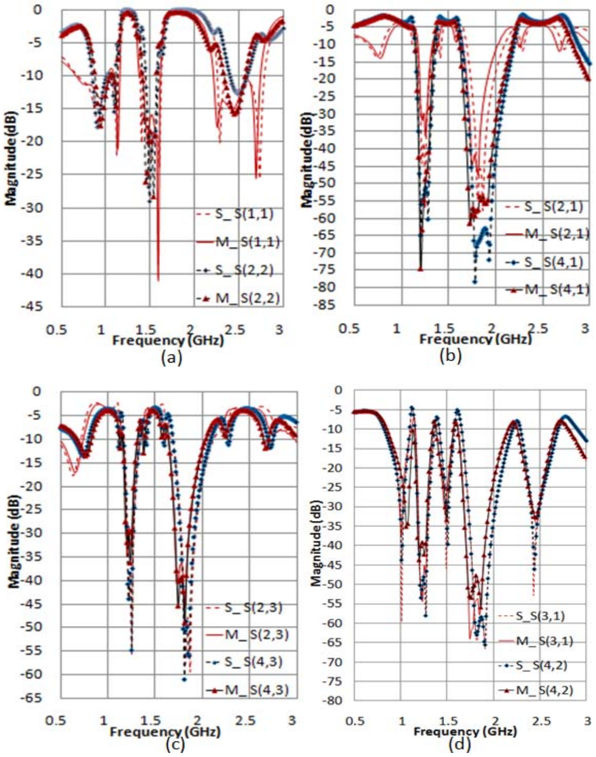


Fig. 5. Topology of proposed tri-band rat-race coupler.

At tri-band 1 /1.5 /2.5 GHz, the measured results in Fig. 6 show that insertion losses are better than 4 dB. The reflection coefficients are better than 10 dB, and the isolation between port 1 and 3, port 2 and 4 are better than 26 dB and 22dB, respectively. As shown in Fig. 7, in phase difference $\angle S_{23}$-$\angle S_{43}$ is less than 2.3°, out-of-phase difference $\angle S_{21}$-$\angle S_{41}$ is less than 183.8°.



(a) (b) (c) (d)

Fig. 6. Simulated (S) and measured (M) results of proposed tri-band rat-race coupler.

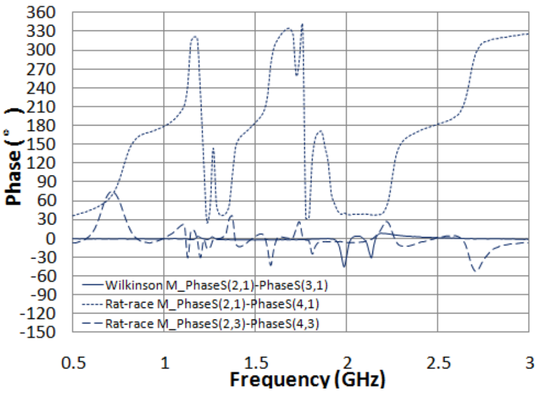


Fig. 7. Measured phase difference of output ports of proposed tri-band Wilkinson power divider and tri-band rat-race coupler.

## IV. CONCLUSION

This paper presents a novel multiband Pi-shaped structure. By adding resonator in conventional Pi-shaped structure to block the signal generated at its corresponding frequency, multiband quarter wavelength transmission line is achieved. Tri-band Wilkinson power divider and tri-band rat-race coupler are designed, fabricated at 1/1.5/2.5 GHz to demonstrate our proposed structure. The excellence performance of our proposed circuits is investigated both by measurement and simulation. Reasonably good agreement is seen. The multiband design method proposed in this paper can be widely applied for multiband applications.

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